

(from slides) Why convolutions?

Neuroscientific inspiration

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

1D convolutions, similar but slightly different than signal processing / math convolutions

[-1, 1] filter/kernel highlights "sharp points" of signal

In [2]:

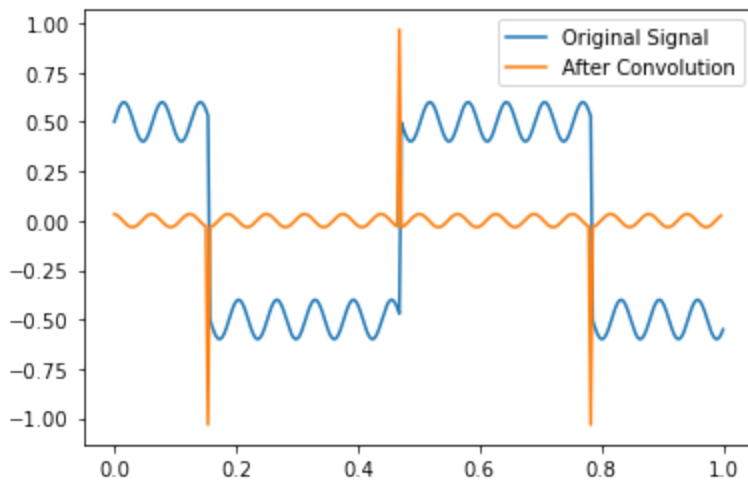
```
import torch
import matplotlib.pyplot as plt
%matplotlib inline

t = torch.linspace(0, 1.0, 300)
x = (torch.cos(10*t) > 0.0).float() + 0.1*torch.sin(100*t)-0.5
plt.plot(t.numpy(), x.numpy(), label='Original Signal')

from torch.nn import functional as F
filt = torch.tensor([-1, 1.0])
print('Filter')
print(filt)
# Should have shape $(m, c, w)$ where $m$ is minibatch size, $c$ is # channels and $w$ is kernel width
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeeze_()
plt.plot(t.numpy()[:len(y)], y.numpy(), label='After Convolution')
plt.legend()
```

```
Filter
tensor([-1.,  1.])
```

Out[2]: <matplotlib.legend.Legend at 0x7f9b2c399dc0>



Convolutions are linear operators (i.e., matrix multiplication) with shared parameters

In [3]:

```
x = torch.randn(10).float().requires_grad_(True)
filt = torch.tensor([-1, 1]).float()
#filt = torch.tensor([1, 2, 3, 4]).float()
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeeze_()

def extract_jacobian(x, y):
    J = torch.zeros((len(y), len(x))).float()
    for i in range(len(y)):
        v = torch.zeros(len(y)).float()
        v[i] = 1
        if x.grad is not None:
            x.grad.zero_()
            y.backward(v, retain_graph=True)
            J[i, :] = x.grad
    return J

A = extract_jacobian(x, y)
print(A)
y2 = torch.matmul(A, x)
print(y)
print(y2)
print(y-y2)
```

```
tensor([[ -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.]])
tensor([-1.4743,  2.6140, -2.9955,  2.4541, -0.6968, -1.3416,  2.0001, -0.3825,
         0.0179], grad_fn=<SqueezeBackward2>)
tensor([-1.4743,  2.6140, -2.9955,  2.4541, -0.6968, -1.3416,  2.0001, -0.3825,
         0.0179], grad_fn=<MvBackward>)
tensor([0., 0., 0., 0., 0., 0., 0., 0., 0.], grad_fn=<SubBackward0>)
```

2D convolutions are similar and can be applied to images

Different filters extract different features from the image

In [8]:

```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2) # Sum channels

filt = torch.tensor([[ -1,  0,  1], [ -1,  0,  1], [ -1,  0,  1]]).float() # Horizontal
filt = torch.tensor([[ -1,  0,  1], [ -1,  0,  1], [ -1,  0,  1]]).float().t() # Vertical
filt = torch.tensor([[ 1, -1], [ -1,  1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()), padding=1)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()), squeeze=True)
print('A size', A.size(), 'B size', B.size())

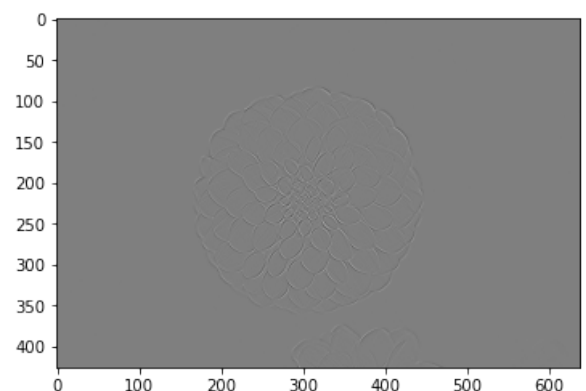
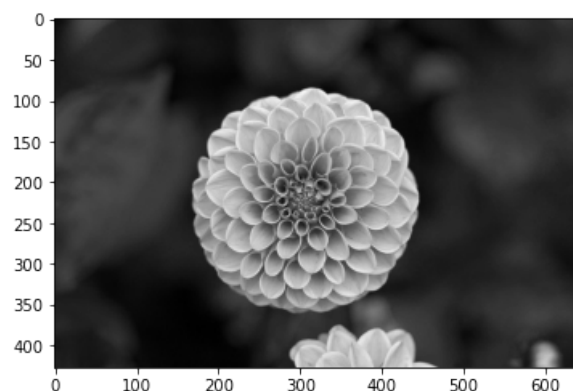
fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.numpy(), cmap='gray')
axes[1].imshow(B.numpy(), cmap='gray')
```

Filter

```
tensor([[ 1., -1.],
        [-1.,  1.]])
```

A size torch.Size([427, 640]) B size torch.Size([426, 639])

Out[8]: <matplotlib.image.AxesImage at 0x7f9b2d4c6a00>



2D convolutions with channel dimension are similar (i.e., if there is more than 1 channel)

In [14]:

```
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = A/255
A = A.permute(2,0,1)
print(A.size())

filt = torch.tensor([1, 0, 0]).reshape(3, 1, 1).float() # Extract red
```

```

filt = torch.tensor([0, 1, 0]).reshape(3, 1, 1).float() # Extract green
filt = torch.tensor([0, 0, 1]).reshape(3, 1, 1).float() # Extract blue
filt = torch.ones(3, 5, 5).float() # Blur
#filt = torch.tensor([
#    [[-1, 1]],
#    [[-1, 1]],
#    [[-1, 1]],
#]).float()

print('Filter')
print(filt)
print(filt.size())
B = F.conv2d(A.reshape(1, *A.size()), filt.reshape(1, *filt.size())).squeeze()
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.permute(1,2,0), cmap='gray')
axes[1].imshow(B, cmap='gray')

```

```
torch.Size([3, 427, 640])
```

```
Filter
```

```

tensor([[[[1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.]],

        [[1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.]],

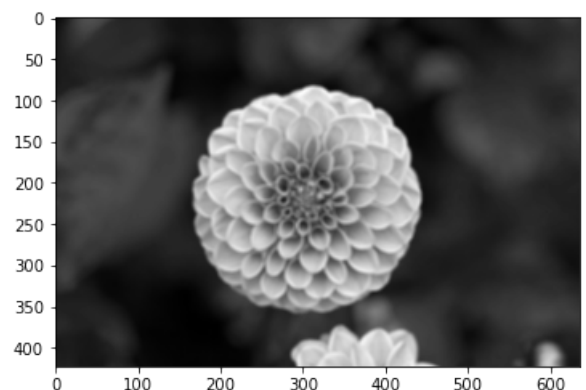
        [[1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.],
          [1., 1., 1., 1., 1.]]]])

```

```
torch.Size([3, 5, 5])
```

```
A size torch.Size([3, 427, 640]) B size torch.Size([423, 636])
```

```
Out[14]: <matplotlib.image.AxesImage at 0x7f9b306c6f10>
```



Need several other components for extracting features: Activation functions and pooling layers

Why activation functions? Activation functions enable non-linear models

Consider a deep linear network

In [5]:

```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.matmul(A2, y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([4.1752])
y2 tensor([4.1752])
```

If you add activation functions, the deep function cannot be simplified

In [6]:

```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.relu(y)
y = torch.matmul(A2, y)
y = torch.relu(y)
y = torch.matmul(A3, y)
print('y', y)

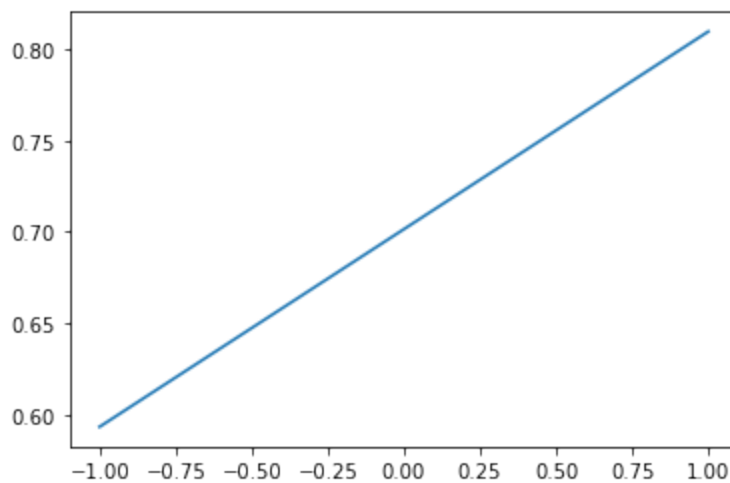
b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([18.9449])
y2 tensor([4.1752])
```

Without ReLU or activation function, the function can only be linear

```
In [23]: N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

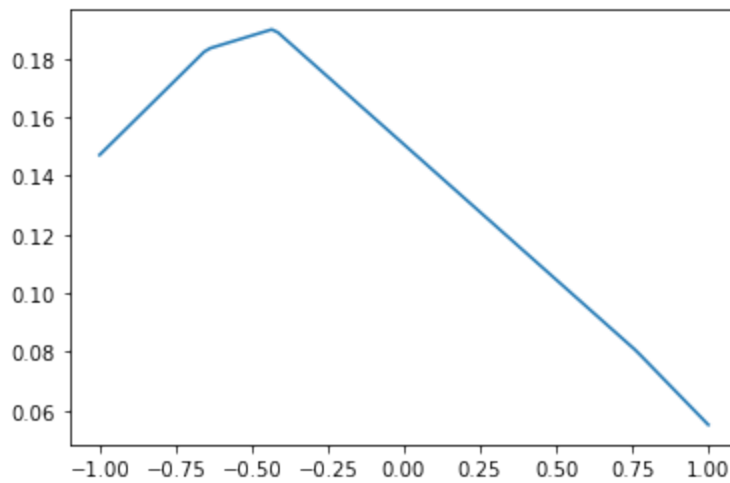
```
Out[23]: [<matplotlib.lines.Line2D at 0x7f9b122d4a30>]
```



With ReLU activation function, the function is *piecewise* linear

```
In [30]: N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.ReLU(),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

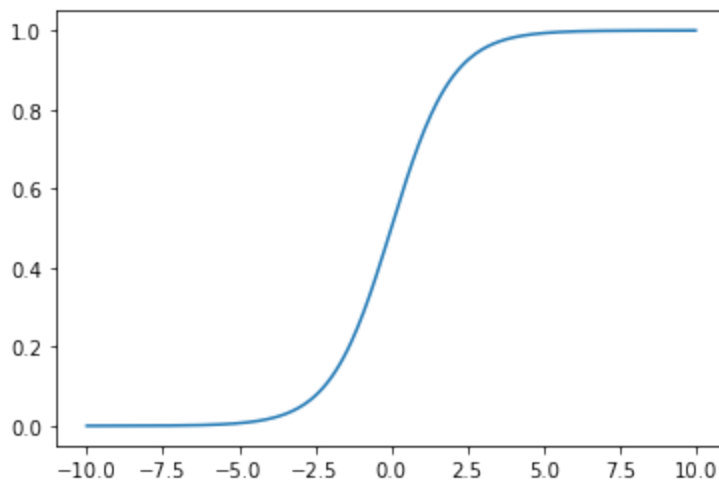
```
Out[30]: [<matplotlib.lines.Line2D at 0x7f9b12e53280>]
```



Common activation functions include sigmoid, ReLU, Leaky ReLU, tanh

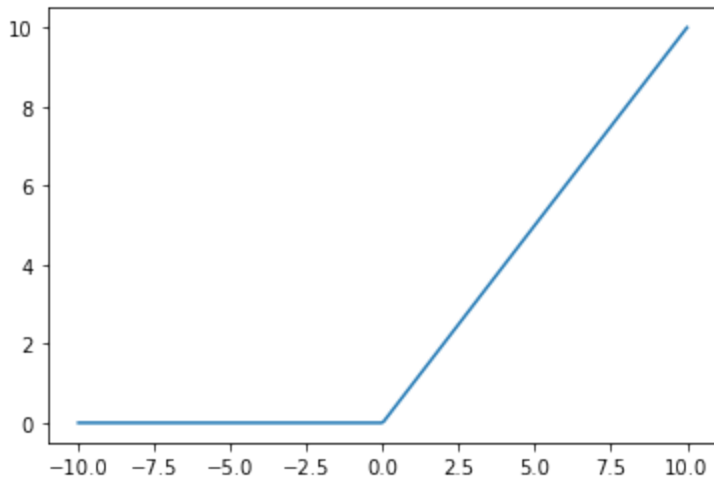
```
In [9]: t = torch.linspace(-10, 10, 300)
plt.plot(t.numpy(), torch.sigmoid(t).numpy())
```

```
Out[9]: [<matplotlib.lines.Line2D at 0x7fb7d92bf850>]
```



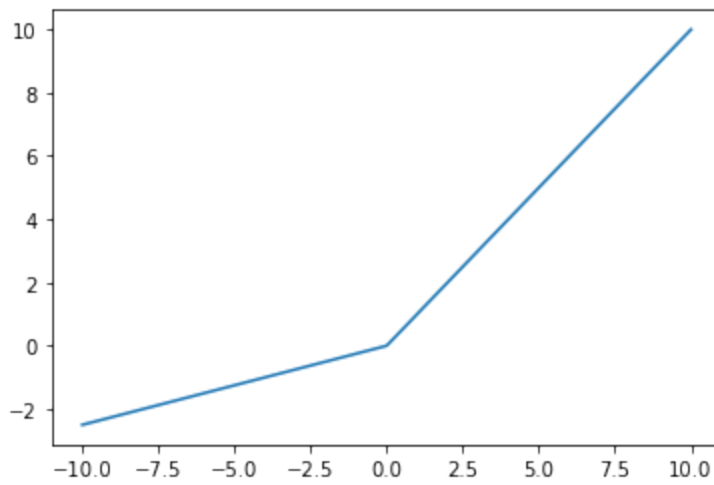
```
In [10]: plt.plot(t.numpy(), F.relu(t).numpy())
```

```
Out[10]: [<matplotlib.lines.Line2D at 0x7fb7d9df4790>]
```



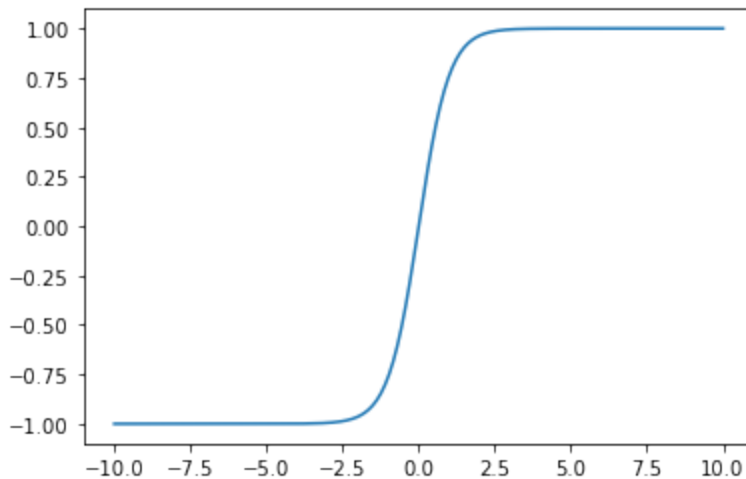
```
In [11]: plt.plot(t.numpy(), F.leaky_relu(t, negative_slope=0.25).numpy())
```

```
Out[11]: [<matplotlib.lines.Line2D at 0x7fb7d9703fd0>]
```



```
In [12]: plt.plot(t.numpy(), torch.tanh(t).numpy())
```

```
Out[12]: [<matplotlib.lines.Line2D at 0x7fb7d97dad90>]
```



Pooling layers are used to reduce dimensionality and introduce some location invariance

Pooling layers include max pooling and average pooling

In [18]:

```
torch.manual_seed(0)
x = torch.randint(10, (10,)).float()
y = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3)
y2 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
y3 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
#y = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3)
#y2 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
#y3 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
print(x)
print(y)
print(y2)
print(y3)
```

```
tensor([4., 9., 3., 0., 3., 9., 7., 3., 7., 3.])
tensor([[9., 9., 7.]])
tensor([[9., 9., 3., 9., 9., 7., 7.]])
tensor([[9., 9., 9., 3., 9., 9., 9., 7., 7.]])
```

In [19]:

```
x = torch.randn((3,4,10,20)).float()
print(x.shape, 'N x C x H x W')
y = F.max_pool2d(x, kernel_size=2)
print(y.shape, 'The number of channels does not change for pooling')
y2 = F.max_pool2d(x, kernel_size=2)
print(y2.shape, 'Note that `stride=kernel_size` by default')
y3 = F.max_pool2d(x, kernel_size=2, stride=1)
print(y3.shape, 'Can set stride explicitly to 1')
y4 = F.max_pool2d(x, kernel_size=3, stride=1, padding=1)
print(y4.shape, 'Can produce the same size')
```

```
torch.Size([3, 4, 10, 20]) N x C x H x W
torch.Size([3, 4, 5, 10]) The number of channels does not change for pooling
torch.Size([3, 4, 5, 10]) Note that `stride=kernel_size` by default
torch.Size([3, 4, 9, 19]) Can set stride explicitly to 1
torch.Size([3, 4, 10, 20]) Can produce the same size
```

Is average pooling a linear or non-linear operation?

Is max pooling a linear or non-linear operation?

Convolution Neural Network (CNN) layers are compositions of convolution, activation and pooling

(See illustration on slide)

In [15]:

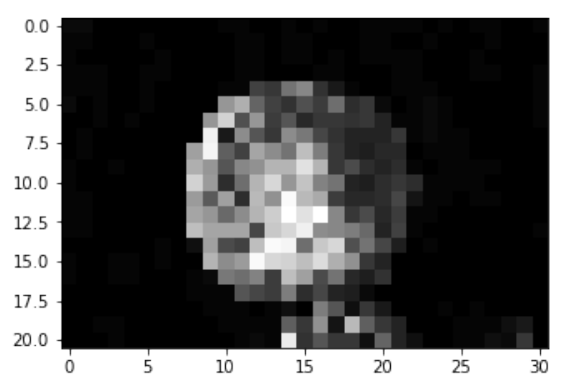
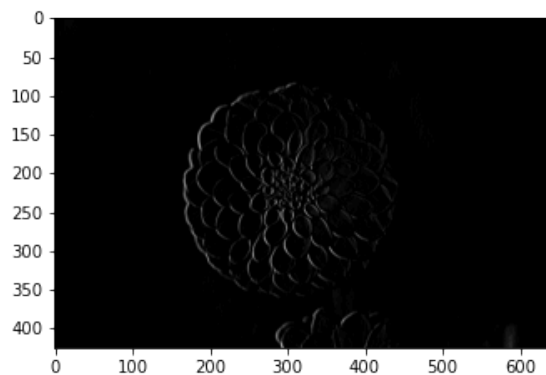
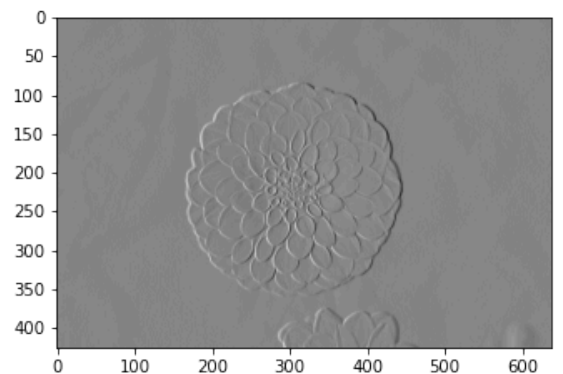
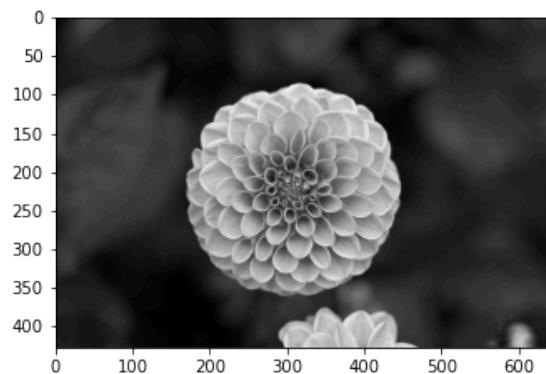
```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2)
filt = torch.tensor([[ -1,  0,  1], [-1,  0,  1], [-1,  0,  1]]).float() # Horizontal
#filt = torch.tensor([[ -1,  0,  1], [-1,  0,  1], [-1,  0,  1]]).float().t() # Vertical
#filt = torch.tensor([[ 1, -1], [-1,  1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()))
print('A size', A.size(), 'B size', B.size())
C = torch.relu(B)
D = torch.max_pool2d(C, kernel_size=20)
#D = torch.max_pool2d(C, kernel_size=20, stride=1)

fig, axes = plt.subplots(2, 2, figsize=(14,8))
axes = axes.ravel()
for im, ax in zip([A, B, C, D], axes):
    ax.imshow(im.squeeze(), cmap='gray')
```

Filter

```
tensor([[ -1.,  0.,  1.],
        [ -1.,  0.,  1.],
        [ -1.,  0.,  1.]])
```

A size torch.Size([427, 640]) B size torch.Size([1, 1, 425, 638])



How could you detect an edge from multiple angles by combining convolutions and ReLUs?

Hint: First detect edges from all directions, then combine.

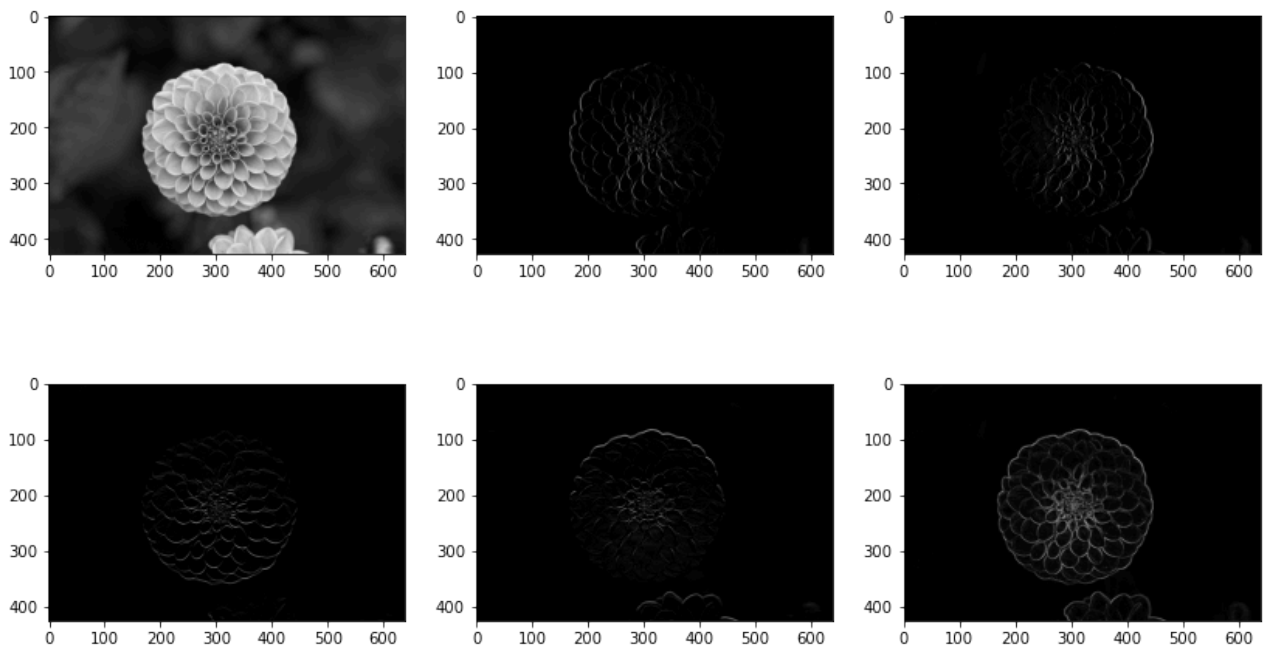
In [16]:

```
import sklearn.datasets
import torch
import numpy as np
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2)

filters = torch.tensor([
    [[[-1, 1], [-1, 1]],
     [[1, -1], [1, -1]],
     [[1, 1], [-1, -1]],
     [[-1, -1], [1, 1]]],
]).float()
B = F.conv2d(A.reshape(1, 1, *A.size()), filters)
C = torch.relu(B)

# Combine
filt = torch.ones(4).float()
D = F.conv2d(C, filt.reshape(1, 4, 1, 1))

fig, axes = plt.subplots(2, 3, figsize=(14,8))
for im, ax in zip([A, *C[0,:,:,:], D], axes.ravel()):
    ax.imshow(im.squeeze(), cmap='gray')
```



Check out PyTorch tutorial on simple classifier on CIFAR10 dataset:

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html